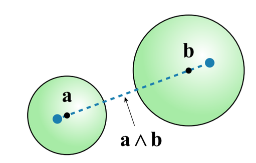
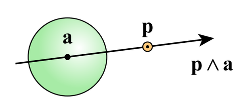
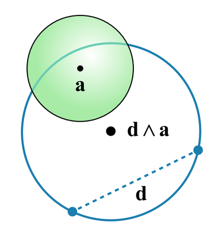
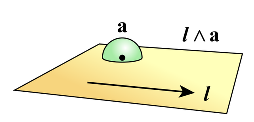
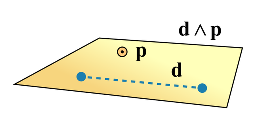
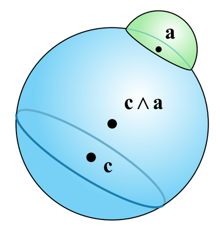
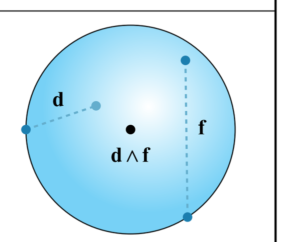
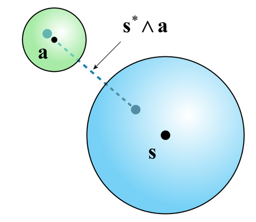
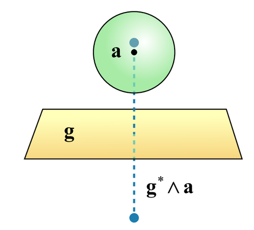
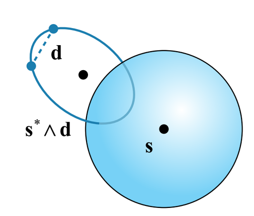
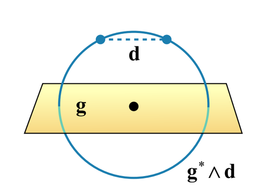
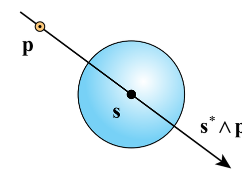
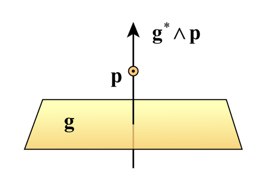
Conformal Geometric Algebra $\mathcal{G}_{4,1}$

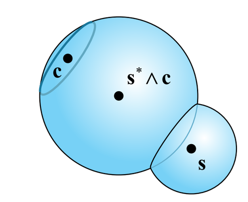
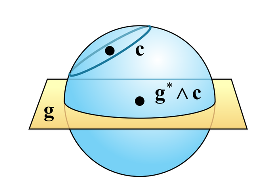
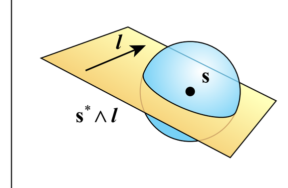
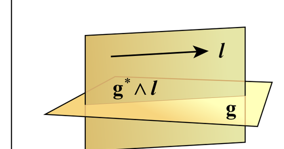
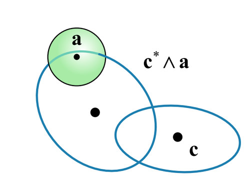
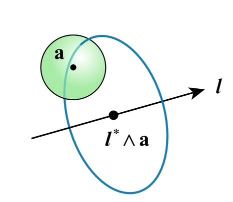
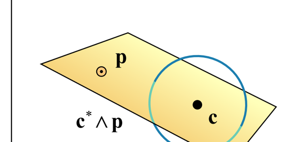
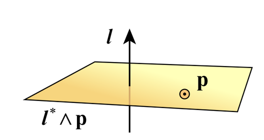
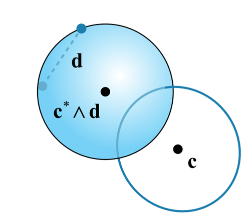
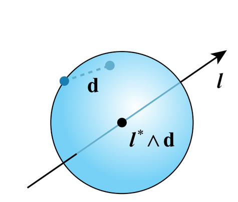
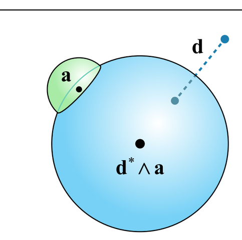
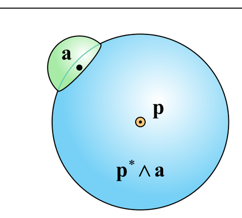
conformalgeometricalgebra.org

JOIN

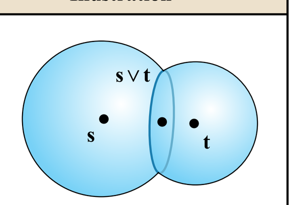
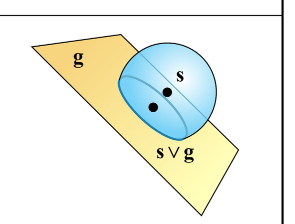
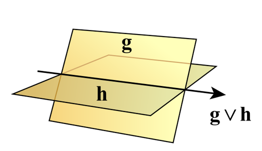
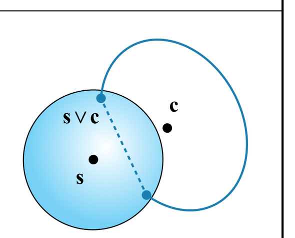
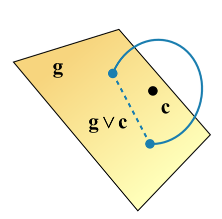
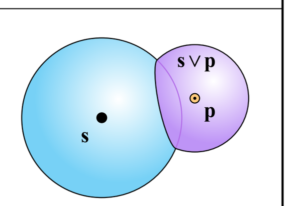
Join Operation	Illustration
Dipole containing round points **a** and **b**. $\mathbf{a}\wedge\mathbf{b} = (a_w b_x - a_x b_w)\mathbf{e}_{41} + (a_w b_y - a_y b_w)\mathbf{e}_{42} + (a_w b_z - a_z b_w)\mathbf{e}_{43} + (a_y b_z - a_z b_y)\mathbf{e}_{23} + (a_z b_x - a_x b_z)\mathbf{e}_{31} + (a_x b_y - a_y b_x)\mathbf{e}_{12} + (a_x b_u - a_u b_x)\mathbf{e}_{15} + (a_y b_u - a_u b_y)\mathbf{e}_{25} + (a_z b_u - a_u b_z)\mathbf{e}_{35} + (a_w b_u - a_u b_w)\mathbf{e}_{45}$	
Line containing flat point **p** and round point **a**. $\mathbf{p}\wedge\mathbf{a} = (p_x a_w - p_w a_x)\mathbf{e}_{415} + (p_z a_y - p_y a_z)\mathbf{e}_{235} + (p_y a_w - p_w a_y)\mathbf{e}_{425} + (p_x a_z - p_z a_x)\mathbf{e}_{315} + (p_z a_w - p_w a_z)\mathbf{e}_{435} + (p_y a_x - p_x a_y)\mathbf{e}_{125}$	
Circle containing dipole **d** and round point **a**. $\mathbf{d}\wedge\mathbf{a} = (d_{vy} a_z - d_{vz} a_y + d_{mx} a_w)\mathbf{e}_{423} + (d_{vz} a_x - d_{vx} a_z + d_{my} a_w)\mathbf{e}_{431} + (d_{vx} a_y - d_{vy} a_x + d_{mz} a_w)\mathbf{e}_{412} - (d_{mx} a_x + d_{my} a_y + d_{mz} a_z)\mathbf{e}_{321} + (d_{px} a_w - d_{pw} a_x + d_{vx} a_u)\mathbf{e}_{415} + (d_{pz} a_y - d_{py} a_z + d_{mx} a_u)\mathbf{e}_{235} + (d_{py} a_w - d_{pw} a_y + d_{vy} a_u)\mathbf{e}_{425} + (d_{px} a_z - d_{pz} a_x + d_{my} a_u)\mathbf{e}_{315} + (d_{pz} a_w - d_{pw} a_z + d_{vz} a_u)\mathbf{e}_{435} + (d_{py} a_x - d_{px} a_y + d_{mz} a_u)\mathbf{e}_{125}$	
Plane containing line ***l*** and round point **a**. $\boldsymbol{l}\wedge\mathbf{a} = (l_{vz} a_y - l_{vy} a_z - l_{mx} a_w)\mathbf{e}_{4235} + (l_{vx} a_z - l_{vz} a_x - l_{my} a_w)\mathbf{e}_{4315} + (l_{vy} a_x - l_{vx} a_y - l_{mz} a_w)\mathbf{e}_{4125} + (l_{mx} a_x + l_{my} a_y + l_{mz} a_z)\mathbf{e}_{3215}$	
Plane containing dipole **d** and flat point **p**. $\mathbf{d}\wedge\mathbf{p} = (d_{vy} p_z - d_{vz} p_y + d_{mx} p_w)\mathbf{e}_{4235} + (d_{vz} p_x - d_{vx} p_z + d_{my} p_w)\mathbf{e}_{4315} + (d_{vx} p_y - d_{vy} p_x + d_{mz} p_w)\mathbf{e}_{4125} - (d_{mx} p_x + d_{my} p_y + d_{mz} p_z)\mathbf{e}_{3215}$	
Sphere containing circle **c** and round point **a**. $\mathbf{c}\wedge\mathbf{a} = -(c_{gx} a_x + c_{gy} a_y + c_{gz} a_z + c_{gw} a_w)\mathbf{e}_{1234} + (c_{vz} a_y - c_{vy} a_z + c_{gx} a_u - c_{mx} a_w)\mathbf{e}_{4235} + (c_{vx} a_z - c_{vz} a_x + c_{gy} a_u - c_{my} a_w)\mathbf{e}_{4315} + (c_{vy} a_x - c_{vx} a_y + c_{gz} a_u - c_{mz} a_w)\mathbf{e}_{4125} + (c_{mx} a_x + c_{my} a_y + c_{mz} a_z + c_{gw} a_u)\mathbf{e}_{3215}$	
Sphere containing dipoles **d** and **f**. $\mathbf{d}\wedge\mathbf{f} = -(d_{vx} f_{mx} + d_{vy} f_{my} + d_{vz} f_{mz} + d_{mx} f_{vx} + d_{my} f_{vy} + d_{mz} f_{vz})\mathbf{e}_{1234} + (d_{vy} f_{pz} - d_{vz} f_{py} + d_{pz} f_{vy} - d_{py} f_{vz} + d_{mx} f_{pw} + d_{pw} f_{mx})\mathbf{e}_{4235} + (d_{vz} f_{px} - d_{vx} f_{pz} + d_{px} f_{vz} - d_{pz} f_{vx} + d_{my} f_{pw} + d_{pw} f_{my})\mathbf{e}_{4315} + (d_{vx} f_{py} - d_{vy} p_{px} + d_{py} f_{vx} - d_{px} f_{vy} + d_{mz} f_{pw} + d_{pw} f_{mz})\mathbf{e}_{4125} - (d_{mx} f_{px} + d_{my} f_{py} + d_{mz} f_{pz} + d_{px} f_{mx} + d_{py} f_{my} + d_{pz} f_{mz})\mathbf{e}_{3215}$	

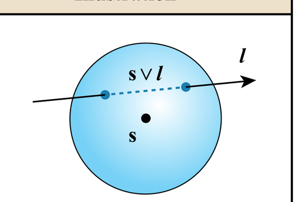
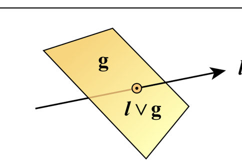
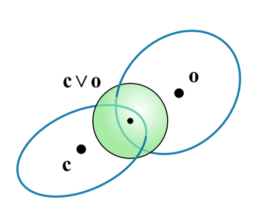
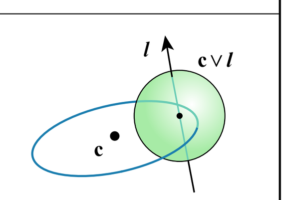
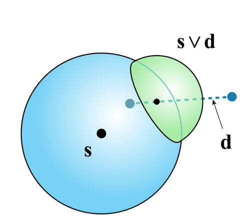
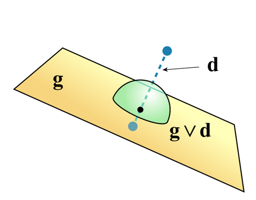
CONNECT

Connect Operation	Illustration
Dipole orthogonal to sphere **s** and containing round point **a**. $\mathbf{s}^*\wedge\mathbf{a} = (s_x a_w + s_u a_x)\mathbf{e}_{41} + (s_z a_y - s_y a_z)\mathbf{e}_{23} + (s_y a_w + s_u a_y)\mathbf{e}_{42} + (s_x a_z - s_z a_x)\mathbf{e}_{31} + (s_z a_w + s_u a_z)\mathbf{e}_{43} + (s_y a_x - s_x a_y)\mathbf{e}_{12} - (s_x a_u + s_w a_x)\mathbf{e}_{15} - (s_y a_u + s_w a_y)\mathbf{e}_{25} - (s_z a_u + s_w a_z)\mathbf{e}_{35} + (s_u a_u - s_w a_w)\mathbf{e}_{45}$	
Dipole orthogonal to plane **g** and containing round point **a**. $\mathbf{g}^*\wedge\mathbf{a} = g_x a_w\mathbf{e}_{41} + (g_z a_y - g_y a_z)\mathbf{e}_{23} + g_y a_w\mathbf{e}_{42} + (g_x a_z - g_z a_x)\mathbf{e}_{31} + g_z a_w\mathbf{e}_{43} + (g_y a_x - g_x a_y)\mathbf{e}_{12} - (g_x a_u + g_w a_x)\mathbf{e}_{15} - (g_y a_u + g_w a_y)\mathbf{e}_{25} - (g_z a_u + g_w a_z)\mathbf{e}_{35} - g_w a_w\mathbf{e}_{45}$	
Circle orthogonal to sphere **s** and containing dipole **d**. $\mathbf{s}^*\wedge\mathbf{d} = (s_y d_{vz} - s_z d_{vy} + s_u d_{mx})\mathbf{e}_{423} + (s_z d_{vx} - s_x d_{vz} + s_u d_{my})\mathbf{e}_{431} + (s_x d_{vy} - s_y d_{vx} + s_u d_{mz})\mathbf{e}_{412} + (s_x d_{mx} + s_y d_{my} + s_z d_{mz})\mathbf{e}_{321} + (s_x d_{pw} + s_w d_{vx} + s_u d_{px})\mathbf{e}_{415} + (s_z d_{py} - s_y d_{pz} + s_w d_{mx})\mathbf{e}_{235} + (s_y d_{pw} + s_w d_{vy} + s_u d_{py})\mathbf{e}_{425} + (s_x d_{pz} - s_z d_{px} + s_w d_{my})\mathbf{e}_{315} + (s_z d_{pw} + s_w d_{vz} + s_u d_{pz})\mathbf{e}_{435} + (s_y d_{px} - s_x d_{py} + s_w d_{mz})\mathbf{e}_{125}$	
Circle orthogonal to plane **g** and containing dipole **d**. $\mathbf{g}^*\wedge\mathbf{d} = (g_y d_{vz} - g_z d_{vy})\mathbf{e}_{423} + (g_z d_{vx} - g_x d_{vz})\mathbf{e}_{431} + (g_x d_{vy} - g_y d_{vx})\mathbf{e}_{412} + (g_x d_{mx} + g_y d_{my} + g_z d_{mz})\mathbf{e}_{321} + (g_x d_{pw} + g_w d_{vx})\mathbf{e}_{415} + (g_z d_{py} - g_y d_{pz} + g_w d_{mx})\mathbf{e}_{235} + (g_y d_{pw} + g_w d_{vy})\mathbf{e}_{425} + (g_x d_{pz} - g_z d_{px} + g_w d_{my})\mathbf{e}_{315} + (g_z d_{pw} + g_w d_{vz})\mathbf{e}_{435} + (g_y d_{px} - g_x d_{py} + g_w d_{mz})\mathbf{e}_{125}$	
Line orthogonal to sphere **s** and containing flat point **p**. $\mathbf{s}^*\wedge\mathbf{p} = (s_x p_w + s_u p_x)\mathbf{e}_{415} + (s_z p_y - s_y p_z)\mathbf{e}_{235} + (s_y p_w + s_u p_y)\mathbf{e}_{425} + (s_x p_z - s_z p_x)\mathbf{e}_{315} + (s_z p_w + s_u p_z)\mathbf{e}_{435} + (s_y p_x - s_x p_y)\mathbf{e}_{125}$	
Line orthogonal to plane **g** and containing flat point **p**. $\mathbf{g}^*\wedge\mathbf{p} = g_x p_w\mathbf{e}_{415} + (g_z p_y - g_y p_z)\mathbf{e}_{235} + g_y p_w\mathbf{e}_{425} + (g_x p_z - g_z p_x)\mathbf{e}_{315} + g_z p_w\mathbf{e}_{435} + (g_y p_x - g_x p_y)\mathbf{e}_{125}$	

Connect Operation	Illustration
Sphere orthogonal to sphere **s** and containing circle **c**. $\mathbf{s}^*\wedge\mathbf{c} = (s_u c_{gw} - s_x c_{gx} - s_y c_{gy} - s_z c_{gz})\mathbf{e}_{1234} + (s_y c_{vz} - s_z c_{vy} - s_w c_{gx} + s_u c_{mx})\mathbf{e}_{4235} + (s_z c_{vx} - s_x c_{vz} - s_w c_{gy} + s_u c_{my})\mathbf{e}_{4315} + (s_x c_{vy} - s_y c_{vx} - s_w c_{gz} + s_u c_{mz})\mathbf{e}_{4125} + (s_x c_{mx} + s_y c_{my} + s_z c_{mz} - s_w c_{gw})\mathbf{e}_{3215}$	
Sphere orthogonal to plane **g** and containing circle **c**. $\mathbf{g}^*\wedge\mathbf{c} = -(g_x c_{gx} + g_y c_{gy} + g_z c_{gz})\mathbf{e}_{1234} + (g_y c_{vz} - g_z c_{vy} - g_w c_{gx})\mathbf{e}_{4235} + (g_z c_{vx} - g_x c_{vz} - g_w c_{gy})\mathbf{e}_{4315} + (g_x c_{vy} - g_y c_{vx} - g_w c_{gz})\mathbf{e}_{4125} + (g_x c_{mx} + g_y c_{my} + g_z c_{mz} - g_w c_{gw})\mathbf{e}_{3215}$	
Plane orthogonal to sphere **s** and containing line ***l***. $\mathbf{s}^*\wedge\boldsymbol{l} = (s_y l_{vz} - s_z l_{vy} + s_u l_{mx})\mathbf{e}_{4235} + (s_z l_{vx} - s_x l_{vz} + s_u l_{my})\mathbf{e}_{4315} + (s_x l_{vy} - s_y l_{vx} + s_u l_{mz})\mathbf{e}_{4125} + (s_x l_{mx} + s_y l_{my} + s_z l_{mz})\mathbf{e}_{3215}$	
Plane orthogonal to plane **g** and containing line ***l***. $\mathbf{g}^*\wedge\boldsymbol{l} = (g_y l_{vz} - g_z l_{vy})\mathbf{e}_{4235} + (g_z l_{vx} - g_x l_{vz})\mathbf{e}_{4315} + (g_x l_{vy} - g_y l_{vx})\mathbf{e}_{4125} + (g_x l_{mx} + g_y l_{my} + g_z l_{mz})\mathbf{e}_{3215}$	
Circle orthogonal to circle **c** and containing round point **a**. $\mathbf{c}^*\wedge\mathbf{a} = (c_{gy} a_z - c_{gz} a_y + c_{vx} a_w)\mathbf{e}_{423} + (c_{gz} a_x - c_{gx} a_z + c_{vy} a_w)\mathbf{e}_{431} + (c_{gx} a_y - c_{gy} a_x + c_{vz} a_w)\mathbf{e}_{412} - (c_{vx} a_x + c_{vy} a_y + c_{vz} a_z)\mathbf{e}_{321} + (c_{mx} a_w + c_{gw} a_x + c_{gx} a_u)\mathbf{e}_{415} + (c_{mz} a_y - c_{my} a_z + c_{vx} a_u)\mathbf{e}_{235} + (c_{my} a_w + c_{gw} a_y + c_{gy} a_u)\mathbf{e}_{425} + (c_{mx} a_z - c_{mz} a_x + c_{vy} a_u)\mathbf{e}_{315} + (c_{mz} a_w + c_{gw} a_z + c_{gz} a_u)\mathbf{e}_{435} + (c_{my} a_x - c_{mx} a_y + c_{vz} a_u)\mathbf{e}_{125}$	
Circle orthogonal to line ***l*** and containing round point **a**. $\boldsymbol{l}^*\wedge\mathbf{a} = l_{vx} a_w\mathbf{e}_{423} + l_{vy} a_w\mathbf{e}_{431} + l_{vz} a_w\mathbf{e}_{412} - (l_{vx} a_x + l_{vy} a_y + l_{vz} a_z)\mathbf{e}_{321} + l_{mx} a_w\mathbf{e}_{415} + (l_{mz} a_y - l_{my} a_z + l_{vx} a_u)\mathbf{e}_{235} + l_{my} a_w\mathbf{e}_{425} + (l_{mx} a_z - l_{mz} a_x + l_{vy} a_u)\mathbf{e}_{315} + l_{mz} a_w\mathbf{e}_{435} + (l_{my} a_x - l_{mx} a_y + l_{vz} a_u)\mathbf{e}_{125}$	
Plane orthogonal to circle **c** and containing flat point **p**. $\mathbf{c}^*\wedge\mathbf{p} = (c_{gy} p_z - c_{gz} p_y + c_{vx} p_w)\mathbf{e}_{4235} + (c_{gz} p_x - c_{gx} p_z + c_{vy} p_w)\mathbf{e}_{4315} + (c_{gx} p_y - c_{gy} p_x + c_{vz} p_w)\mathbf{e}_{4125} - (c_{vx} p_x + c_{vy} p_y + c_{vz} p_z)\mathbf{e}_{3215}$	
Plane orthogonal to line ***l*** and containing flat point **p**. $\boldsymbol{l}^*\wedge\mathbf{p} = l_{vx} p_w\mathbf{e}_{4235} + l_{vy} p_w\mathbf{e}_{4315} + l_{vz} p_w\mathbf{e}_{4125} - (l_{vx} p_x + l_{vy} p_y + l_{vz} p_z)\mathbf{e}_{3215}$	
Sphere orthogonal to circle **c** and containing dipole **d**. $\mathbf{c}^*\wedge\mathbf{d} = -(c_{vx} d_{vx} + c_{vy} d_{vy} + c_{vz} d_{vz} + c_{gx} d_{mx} + c_{gy} d_{my} + c_{gz} d_{mz})\mathbf{e}_{1234} + (c_{mz} d_{vy} - c_{my} d_{vz} + c_{vx} d_{pw} + c_{gy} d_{pz} - c_{gz} d_{py} - c_{gw} d_{mx})\mathbf{e}_{4235} + (c_{mx} d_{vz} - c_{mz} d_{vx} + c_{vy} d_{pw} + c_{gz} d_{px} - c_{gx} d_{pz} - c_{gw} d_{my})\mathbf{e}_{4315} + (c_{my} d_{vx} - c_{mx} d_{vy} + c_{vz} d_{pw} + c_{gx} d_{py} - c_{gy} d_{px} - c_{gw} d_{mz})\mathbf{e}_{4125} - (c_{vx} d_{px} + c_{vy} d_{py} + c_{vz} d_{pz} + c_{mx} d_{mx} + c_{my} d_{my} + c_{mz} d_{mz})\mathbf{e}_{3215}$	
Sphere orthogonal to line ***l*** and containing dipole **d**. $\boldsymbol{l}^*\wedge\mathbf{d} = -(l_{vx} d_{vx} + l_{vy} d_{vy} + l_{vz} d_{vz})\mathbf{e}_{1234} + (l_{mz} d_{vy} - l_{my} d_{vz} + l_{vx} d_{pw})\mathbf{e}_{4235} + (l_{mx} d_{vz} - l_{mz} d_{vx} + l_{vy} d_{pw})\mathbf{e}_{4315} + (l_{my} d_{vx} - l_{mx} d_{vy} + l_{vz} d_{pw})\mathbf{e}_{4125} - (l_{vx} d_{px} + l_{vy} d_{py} + l_{vz} d_{pz} + l_{mx} d_{mx} + l_{my} d_{my} + l_{mz} d_{mz})\mathbf{e}_{3215}$	
Sphere orthogonal to dipole **d** and containing round point **a**. $\mathbf{d}^*\wedge\mathbf{a} = (d_{vx} a_x + d_{vy} a_y + d_{vz} a_z - d_{pw} a_w)\mathbf{e}_{1234} + (d_{my} a_z - d_{mz} a_y + d_{px} a_w - d_{vx} a_u)\mathbf{e}_{4235} + (d_{mz} a_x - d_{mx} a_z + d_{py} a_w - d_{vy} a_u)\mathbf{e}_{4315} + (d_{mx} a_y - d_{my} a_x + d_{pz} a_w - d_{vz} a_u)\mathbf{e}_{4125} + (d_{pw} a_u - d_{px} a_x - d_{py} a_y - d_{pz} a_z)\mathbf{e}_{3215}$	
Sphere centered at flat point **p** and containing round point **a**. $\mathbf{p}^*\wedge\mathbf{a} = -p_w a_w\mathbf{e}_{1234} + p_x a_w\mathbf{e}_{4235} + p_y a_w\mathbf{e}_{4315} + p_z a_w\mathbf{e}_{4125} + (p_w a_u - p_x a_x - p_y a_y - p_z a_z)\mathbf{e}_{3215}$	

MEET

Meet Operation	Illustration
Circle where spheres **s** and **t** intersect. $\mathbf{s}\vee\mathbf{t} = (s_u t_x - s_x t_u)\mathbf{e}_{423} + (s_u t_y - s_y t_u)\mathbf{e}_{431} + (s_u t_z - s_z t_u)\mathbf{e}_{412} + (s_u t_w - s_w t_u)\mathbf{e}_{321} + (s_z t_y - s_y t_z)\mathbf{e}_{415} + (s_x t_z - s_z t_x)\mathbf{e}_{425} + (s_y t_x - s_x t_y)\mathbf{e}_{435} + (s_x t_w - s_w t_x)\mathbf{e}_{235} + (s_y t_w - s_w t_y)\mathbf{e}_{315} + (s_z t_w - s_w t_z)\mathbf{e}_{125}$	
Circle where sphere **s** and plane **g** intersect. $\mathbf{s}\vee\mathbf{g} = s_u g_x\mathbf{e}_{423} + s_u g_y\mathbf{e}_{431} + s_u g_z\mathbf{e}_{412} + s_u g_w\mathbf{e}_{321} + (s_z g_y - s_y g_z)\mathbf{e}_{415} + (s_x g_z - s_z g_x)\mathbf{e}_{425} + (s_y g_x - s_x g_y)\mathbf{e}_{435} + (s_x g_w - s_w g_x)\mathbf{e}_{235} + (s_y g_w - s_w g_y)\mathbf{e}_{315} + (s_z g_w - s_w g_z)\mathbf{e}_{125}$	
Line where planes **g** and **h** intersect. $\mathbf{g}\vee\mathbf{h} = (g_z h_y - g_y h_z)\mathbf{e}_{415} + (g_x h_w - g_w h_x)\mathbf{e}_{235} + (g_x h_z - g_z h_x)\mathbf{e}_{425} + (g_y h_w - g_w h_y)\mathbf{e}_{315} + (g_y h_x - g_x h_y)\mathbf{e}_{435} + (g_z h_w - g_w h_z)\mathbf{e}_{125}$	
Dipole where sphere **s** and circle **c** intersect. $\mathbf{s}\vee\mathbf{c} = (s_y c_{gz} - s_z c_{gy} + s_u c_{vx})\mathbf{e}_{41} + (s_w c_{gx} - s_x c_{gw} + s_u c_{mx})\mathbf{e}_{23} + (s_z c_{gx} - s_x c_{gz} + s_u c_{vy})\mathbf{e}_{42} + (s_w c_{gy} - s_y c_{gw} + s_u c_{my})\mathbf{e}_{31} + (s_x c_{gy} - s_y c_{gx} + s_u c_{vz})\mathbf{e}_{43} + (s_w c_{gz} - s_z c_{gw} + s_u c_{mz})\mathbf{e}_{12} + (s_z c_{my} - s_y c_{mz} + s_w c_{vx})\mathbf{e}_{15} + (s_x c_{mz} - s_z c_{mx} + s_w c_{vy})\mathbf{e}_{25} + (s_y c_{mx} - s_x c_{my} + s_w c_{vz})\mathbf{e}_{35} - (s_x c_{vx} + s_y c_{vy} + s_z c_{vz})\mathbf{e}_{45}$	
Dipole where plane **g** and circle **c** intersect. $\mathbf{g}\vee\mathbf{c} = (g_y c_{gz} - g_z c_{gy})\mathbf{e}_{41} + (g_w c_{gx} - g_x c_{gw})\mathbf{e}_{23} + (g_z c_{gx} - g_x c_{gz})\mathbf{e}_{42} + (g_w c_{gy} - g_y c_{gw})\mathbf{e}_{31} + (g_x c_{gy} - g_y c_{gx})\mathbf{e}_{43} + (g_w c_{gz} - g_z c_{gw})\mathbf{e}_{12} + (g_z c_{my} - g_y c_{mz} + g_w c_{vx})\mathbf{e}_{15} + (g_x c_{mz} - g_z c_{mx} + g_w c_{vy})\mathbf{e}_{25} + (g_y c_{mx} - g_x c_{my} + g_w c_{vz})\mathbf{e}_{35} - (g_x c_{vx} + g_y c_{vy} + g_z c_{vz})\mathbf{e}_{45}$	
Round point centered at flat point **p** and contained by sphere **s**. $\mathbf{s}\vee\mathbf{p} = s_u p_x\mathbf{e}_1 + s_u p_y\mathbf{e}_2 + s_u p_z\mathbf{e}_3 + s_u p_w\mathbf{e}_4 - (s_x p_x + s_y p_y + s_z p_z + s_w p_w)\mathbf{e}_5$	

Meet Operation	Illustration
Dipole where sphere **s** and line ***l*** intersect. $\mathbf{s}\vee\boldsymbol{l} = s_u l_{vx}\mathbf{e}_{41} + s_u l_{vy}\mathbf{e}_{42} + s_u l_{vz}\mathbf{e}_{43} + s_u l_{mx}\mathbf{e}_{23} + s_u l_{my}\mathbf{e}_{31} + s_u l_{mz}\mathbf{e}_{12} + (s_z l_{my} - s_y l_{mz} + s_w l_{vx})\mathbf{e}_{15} + (s_x l_{mz} - s_z l_{mx} + s_w l_{vy})\mathbf{e}_{25} + (s_y l_{mx} - s_x l_{my} + s_w l_{vz})\mathbf{e}_{35} - (s_x l_{vx} + s_y l_{vy} + s_z l_{vz})\mathbf{e}_{45}$	
Flat point where line ***l*** intersects plane **g**. $\mathbf{g}\vee\boldsymbol{l} = (g_z l_{my} - g_y l_{mz} + g_w l_{vx})\mathbf{e}_{15} + (g_x l_{mz} - g_z l_{mx} + g_w l_{vy})\mathbf{e}_{25} + (g_y l_{mx} - g_x l_{my} + g_w l_{vz})\mathbf{e}_{35} - (g_x l_{vx} + g_y l_{vy} + g_z l_{vz})\mathbf{e}_{45}$	
Round point contained by circles **c** and **o**. $\mathbf{c}\vee\mathbf{o} = (c_{gz} o_{my} - c_{gy} o_{mz} + c_{my} o_{gz} - c_{mz} o_{gy} + c_{vx} o_{gw} + c_{gw} o_{vx})\mathbf{e}_1 + (c_{gx} o_{mz} - c_{gz} o_{mx} + c_{mz} o_{gx} - c_{mx} o_{gz} + c_{vy} o_{gw} + c_{gw} o_{vy})\mathbf{e}_2 + (c_{gy} o_{mx} - c_{gx} o_{my} + c_{mx} o_{gy} - c_{my} o_{gx} + c_{vz} o_{gw} + c_{gw} o_{vz})\mathbf{e}_3 - (c_{gx} o_{vx} + c_{gy} o_{vy} + c_{gz} o_{vz} + c_{vx} o_{gx} + c_{vy} o_{gy} + c_{vz} o_{gz})\mathbf{e}_4 - (c_{mx} o_{vx} + c_{my} o_{vy} + c_{mz} o_{vz} + c_{vx} o_{mx} + c_{vy} o_{my} + c_{vz} o_{mz})\mathbf{e}_5$	
Round point centered on line ***l*** and contained by circle **c**. $\mathbf{c}\vee\boldsymbol{l} = (c_{gz} l_{my} - c_{gy} l_{mz} + c_{gw} l_{vx})\mathbf{e}_1 + (c_{gx} l_{mz} - c_{gz} l_{mx} + c_{gw} l_{vy})\mathbf{e}_2 + (c_{gy} l_{mx} - c_{gx} l_{my} + c_{gw} l_{vz})\mathbf{e}_3 - (c_{gx} l_{vx} + c_{gy} l_{vy} + c_{vz} l_{vz})\mathbf{e}_4 - (c_{mx} l_{vx} + c_{my} l_{vy} + c_{mz} l_{vz} + c_{vx} l_{mx} + c_{vy} l_{my} + c_{vz} l_{mz})\mathbf{e}_5$	
Round point contained by sphere **s** and dipole **d**. $\mathbf{s}\vee\mathbf{d} = (s_y d_{mz} - s_z d_{my} - s_w d_{vx} + s_u d_{px})\mathbf{e}_1 + (s_z d_{mx} - s_x d_{mz} - s_w d_{vy} + s_u d_{py})\mathbf{e}_2 + (s_x d_{my} - s_y d_{mx} - s_w d_{vz} + s_u d_{pz})\mathbf{e}_3 + (s_x d_{vx} + s_y d_{vy} + s_z d_{vz} + s_u d_{pw})\mathbf{e}_4 - (s_x d_{px} + s_y d_{py} + s_z d_{pz} + s_w d_{pw})\mathbf{e}_5$	
Round point centered in plane **g** and contained by dipole **d**. $\mathbf{g}\vee\mathbf{d} = (g_y d_{mz} - g_z d_{my} - g_w d_{vx})\mathbf{e}_1 + (g_z d_{mx} - g_x d_{mz} - g_w d_{vy})\mathbf{e}_2 + (g_x d_{my} - g_y d_{mx} - g_w d_{vz})\mathbf{e}_3 + (g_x d_{vx} + g_y d_{vy} + g_z d_{vz})\mathbf{e}_4 - (g_x d_{px} + g_y d_{py} + g_z d_{pz} + g_w d_{pw})\mathbf{e}_5$	

FLATS

Flat Point p (Bivector) 0D

$$\mathbf{p} = p_x\mathbf{e}_{15} + p_y\mathbf{e}_{25} + p_z\mathbf{e}_{35} + p_w\mathbf{e}_{45}$$

Dual	$\mathbf{p}^* = -p_x\mathbf{e}_{235} - p_y\mathbf{e}_{315} - p_z\mathbf{e}_{125} + p_w\mathbf{e}_{321}$
Attitude	$\operatorname{att}(\mathbf{p}) = \mathbf{p}\vee\mathbf{e}_{3215} = p_w\mathbf{e}_5$
Flat Bulk	$\mathbf{p}_\blacksquare = p_x\mathbf{e}_{15} + p_y\mathbf{e}_{25} + p_z\mathbf{e}_{35}$
Flat Weight	$\mathbf{p}_\square = p_w\mathbf{e}_{45}$
Position Norm	$\dfrac{\lVert\mathbf{p}\rVert_\blacksquare}{\lVert\mathbf{p}\rVert_\square} = \sqrt{\dfrac{p_x^2 + p_y^2 + p_z^2}{p_w^2}}$

Flat Line *l* (Trivector) 1D

$$\boldsymbol{l} = l_{vx}\mathbf{e}_{415} + l_{vy}\mathbf{e}_{425} + l_{vz}\mathbf{e}_{435} + l_{mx}\mathbf{e}_{235} + l_{my}\mathbf{e}_{315} + l_{mz}\mathbf{e}_{125}$$

$$l_{vx}l_{mx} + l_{vy}l_{my} + l_{vz}l_{mz} = 0$$

Dual	$\boldsymbol{l}^* = l_{vx}\mathbf{e}_{23} + l_{vy}\mathbf{e}_{31} + l_{vz}\mathbf{e}_{12} + l_{mx}\mathbf{e}_{15} + l_{my}\mathbf{e}_{25} + l_{mz}\mathbf{e}_{35}$
Attitude	$\operatorname{att}(\boldsymbol{l}) = \boldsymbol{l}\vee\mathbf{e}_{3215} = l_{vx}\mathbf{e}_{15} + l_{vy}\mathbf{e}_{25} + l_{vz}\mathbf{e}_{35}$
Flat Bulk	$\boldsymbol{l}_\blacksquare = l_{mx}\mathbf{e}_{235} + l_{my}\mathbf{e}_{315} + l_{mz}\mathbf{e}_{125}$ (moment)
Flat Weight	$\boldsymbol{l}_\square = l_{vx}\mathbf{e}_{415} + l_{vy}\mathbf{e}_{425} + l_{vz}\mathbf{e}_{435}$ (direction)
Position Norm	$\dfrac{\lVert\boldsymbol{l}\rVert_\blacksquare}{\lVert\boldsymbol{l}\rVert_\square} = \sqrt{\dfrac{l_{mx}^2 + l_{my}^2 + l_{mz}^2}{l_{vx}^2 + l_{vy}^2 + l_{vz}^2}}$

Flat Plane g (Quadrivector) 2D

$$\mathbf{g} = g_x\mathbf{e}_{4235} + g_y\mathbf{e}_{4315} + g_z\mathbf{e}_{4125} + g_w\mathbf{e}_{3215}$$

Dual	$\mathbf{g}^* = -g_x\mathbf{e}_1 - g_y\mathbf{e}_2 - g_z\mathbf{e}_3 + g_w\mathbf{e}_5$
Attitude	$\operatorname{att}(\mathbf{g}) = \mathbf{g}\vee\mathbf{e}_{3215} = g_x\mathbf{e}_{235} + g_y\mathbf{e}_{315} + g_z\mathbf{e}_{125}$
Flat Bulk	$\mathbf{g}_\blacksquare = g_w\mathbf{e}_{3215}$ (position)
Flat Weight	$\mathbf{g}_\square = g_x\mathbf{e}_{4235} + g_y\mathbf{e}_{4315} + g_z\mathbf{e}_{4125}$ (normal)
Position Norm	$\dfrac{\lVert\mathbf{g}\rVert_\blacksquare}{\lVert\mathbf{g}\rVert_\square} = \sqrt{\dfrac{g_w^2}{g_x^2 + g_y^2 + g_z^2}}$

ROUNDS

Round Point a (Vector) 0D

$r^2 > 0$

$r^2 < 0$

$$\mathbf{a} = p_x\mathbf{e}_1 + p_y\mathbf{e}_2 + p_z\mathbf{e}_3 + \mathbf{e}_4 + \frac{p^2 + r^2}{2}\mathbf{e}_5$$

p = position, r = radius

$$\mathbf{a} = \underbrace{a_x\mathbf{e}_1 + a_y\mathbf{e}_2 + a_z\mathbf{e}_3 + a_w\mathbf{e}_4}_{\text{Carrier Point}} + \underbrace{a_u\mathbf{e}_5}_{\text{Infinity}}$$

Infinity (when $a_x = a_y = a_z = a_w = 0$)

Attitude: $\operatorname{att}(\mathbf{a}) = \mathbf{a}\vee\mathbf{e}_{3215} = a_w\mathbf{1}$

Center	$\operatorname{cen}(\mathbf{a}) = -\operatorname{acr}(\mathbf{a})\vee\mathbf{a} = a_x a_w\mathbf{e}_1 + a_y a_w\mathbf{e}_2 + a_z a_w\mathbf{e}_3 + a_w^2\mathbf{e}_4 + a_w a_u\mathbf{e}_5$ (identity)		
Container	$\operatorname{con}(\mathbf{a}) = \operatorname{car}(\mathbf{a})^*\wedge\mathbf{a} = -a_w^2\mathbf{e}_{1234} + a_x a_w\mathbf{e}_{4235} + a_y a_w\mathbf{e}_{4315} + a_z a_w\mathbf{e}_{4125} + (a_w a_u - a_x^2 - a_y^2 - a_z^2)\mathbf{e}_{3215}$		
Dual	$\mathbf{a}^* = -a_w\mathbf{e}_{1234} + a_x\mathbf{e}_{4235} + a_y\mathbf{e}_{4315} + a_z\mathbf{e}_{4125} - a_u\mathbf{e}_{3215}$		
Carrier	$\operatorname{car}(\mathbf{a}) = \mathbf{a}\wedge\mathbf{e}_5 = a_x\mathbf{e}_{15} + a_y\mathbf{e}_{25} + a_z\mathbf{e}_{35} + a_w\mathbf{e}_{45}$ (flat point)		
Anticarrier	$\operatorname{acr}(\mathbf{a}) = \mathbf{a}^*\wedge\mathbf{e}_5 = -a_w\mathbb{1}$ (volume)		
Round Bulk	$\mathbf{a}_\bullet = a_x\mathbf{e}_1 + a_y\mathbf{e}_2 + a_z\mathbf{e}_3$	Center Norm	$\dfrac{\lVert\mathbf{a}\rVert_\odot}{\lVert\mathbf{a}\rVert_\circ} = \sqrt{\dfrac{a_x^2 + a_y^2 + a_z^2}{a_w^2}}$
Round Weight	$\mathbf{a}_\circ = a_w\mathbf{e}_4$		
Flat Bulk	$\mathbf{a}_\blacksquare = a_u\mathbf{e}_5$	Radius Norm	$\dfrac{\lVert\mathbf{a}\rVert_\odot}{\lVert\mathbf{a}\rVert_\circ} = \sqrt{\dfrac{2a_w a_u - a_x^2 - a_y^2 - a_z^2}{a_w^2}}$
Flat Weight	$\mathbf{a}_\square = 0$		

Sphere s (Quadrivector) 3D

$$\mathbf{s} = p_x\mathbf{e}_{4235} + p_y\mathbf{e}_{4315} + p_z\mathbf{e}_{4125} - \mathbf{e}_{1234} - \frac{p^2 - r^2}{2}\mathbf{e}_{3215}$$

p = center, r = radius

$$\mathbf{s} = \underbrace{s_u\mathbf{e}_{1234}}_{\text{Carrier Space}} + \underbrace{s_x\mathbf{e}_{4235} + s_y\mathbf{e}_{4315} + s_z\mathbf{e}_{4125} + s_w\mathbf{e}_{3215}}_{\text{Flat Plane}}$$

Flat Plane (when $s_u = 0$)

Attitude: $\operatorname{att}(\mathbf{s}) = \mathbf{s}\vee\mathbf{e}_{3215} = s_u\mathbf{e}_{321} + s_x\mathbf{e}_{235} + s_y\mathbf{e}_{315} + s_z\mathbf{e}_{125}$

Center	$\operatorname{cen}(\mathbf{s}) = -\operatorname{acr}(\mathbf{s})\vee\mathbf{s} = -s_x s_u\mathbf{e}_1 - s_y s_u\mathbf{e}_2 - s_z s_u\mathbf{e}_3 + s_u^2\mathbf{e}_4 + (s_x^2 + s_y^2 + s_z^2 - s_w s_u)\mathbf{e}_5$		
Container	$\operatorname{con}(\mathbf{s}) = \operatorname{car}(\mathbf{s})^*\wedge\mathbf{s} = -s_u^2\mathbf{e}_{1234} - s_x s_u\mathbf{e}_{4235} - s_y s_u\mathbf{e}_{4315} - s_z s_u\mathbf{e}_{4125} - s_w s_u\mathbf{e}_{3215}$ (identity)		
Dual	$\mathbf{s}^* = -s_x\mathbf{e}_1 - s_y\mathbf{e}_2 - s_z\mathbf{e}_3 + s_u\mathbf{e}_4 + s_w\mathbf{e}_5$		
Carrier	$\operatorname{car}(\mathbf{s}) = \mathbf{s}\wedge\mathbf{e}_5 = s_u\mathbb{1}$ (volume)		
Anticarrier	$\operatorname{acr}(\mathbf{s}) = \mathbf{s}^*\wedge\mathbf{e}_5 = -s_x\mathbf{e}_{15} - s_y\mathbf{e}_{25} - s_z\mathbf{e}_{35} + s_u\mathbf{e}_{45}$ (flat point)		
Round Bulk	$\mathbf{s}_\bullet = 0$	Center Norm	$\dfrac{\lVert\mathbf{s}\rVert_\odot}{\lVert\mathbf{s}\rVert_\circ} = \sqrt{\dfrac{s_x^2 + s_y^2 + s_z^2}{s_u^2}}$
Round Weight	$\mathbf{s}_\circ = s_u\mathbf{e}_{1234}$		
Flat Bulk	$\mathbf{s}_\blacksquare = s_w\mathbf{e}_{3215}$	Radius Norm	$\dfrac{\lVert\mathbf{s}\rVert_\odot}{\lVert\mathbf{s}\rVert_\circ} = \sqrt{\dfrac{s_x^2 + s_y^2 + s_z^2 - 2s_w s_u}{s_u^2}}$
Flat Weight	$\mathbf{s}_\square = s_x\mathbf{e}_{4235} + s_y\mathbf{e}_{4315} + s_z\mathbf{e}_{4125}$		

Dipole d (Bivector) 1D

$$\mathbf{d} = n_x\mathbf{e}_{41} + n_y\mathbf{e}_{42} + n_z\mathbf{e}_{43} + (p_y n_z - p_z n_y)\mathbf{e}_{23} + (p_z n_x - p_x n_z)\mathbf{e}_{31} + (p_x n_y - p_y n_x)\mathbf{e}_{12} + (\mathbf{p}\cdot\mathbf{n})(p_x\mathbf{e}_{15} + p_y\mathbf{e}_{25} + p_z\mathbf{e}_{35} + \mathbf{e}_{45}) - \frac{p^2 + r^2}{2}(n_x\mathbf{e}_{15} + n_y\mathbf{e}_{25} + n_z\mathbf{e}_{35})$$

$\mathbf{p} = (p_x, p_y, p_z)$ = center

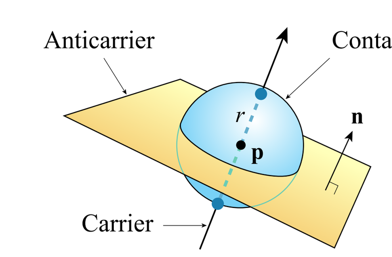
$\mathbf{n} = (n_x, n_y, n_z)$ = direction

r = radius

Anticarrier Normal; Anticarrier Position

$$\mathbf{d} = \underbrace{d_{vx}\mathbf{e}_{41} + d_{vy}\mathbf{e}_{42} + d_{vz}\mathbf{e}_{43} + d_{mx}\mathbf{e}_{23} + d_{my}\mathbf{e}_{31} + d_{mz}\mathbf{e}_{12}}_{\text{Carrier Line}} + \underbrace{d_{px}\mathbf{e}_{15} + d_{py}\mathbf{e}_{25} + d_{pz}\mathbf{e}_{35} + d_{pw}\mathbf{e}_{45}}_{\text{Flat Point}}$$

Flat Point (when $d_{vz} = d_{vy} = d_{vz} = d_{mx} = d_{my} = d_{mz} = 0$)



$$d_{py}d_{vz} - d_{pz}d_{vy} - d_{pw}d_{mx} = 0$$
$$d_{pz}d_{vx} - d_{px}d_{vz} - d_{pw}d_{my} = 0$$
$$d_{px}d_{vy} - d_{py}d_{vx} - d_{pw}d_{mz} = 0$$
$$d_{px}d_{mx} + d_{py}d_{my} + d_{pz}d_{mz} = 0$$
$$d_{vx}d_{mx} + d_{vy}d_{my} + d_{vz}d_{mz} = 0$$

Center	$\operatorname{cen}(\mathbf{d}) = -\operatorname{acr}(\mathbf{d})\vee\mathbf{d} = (d_{vy}d_{mz} - d_{vz}d_{my} + d_{vx}d_{pw})\mathbf{e}_1 + (d_{vz}d_{mx} - d_{vx}d_{mz} + d_{vy}d_{pw})\mathbf{e}_2 + (d_{vx}d_{my} - d_{vy}d_{mx} + d_{vz}d_{pw})\mathbf{e}_3 + (d_{vx}^2 + d_{vy}^2 + d_{vz}^2)\mathbf{e}_4 + (d_{pw}^2 - d_{vx}d_{px} - d_{vy}d_{py} - d_{vz}d_{pz})\mathbf{e}_5$
Dual	$\mathbf{d}^* = -d_{vx}\mathbf{e}_{423} - d_{vy}\mathbf{e}_{431} - d_{vz}\mathbf{e}_{412} + d_{pw}\mathbf{e}_{321} - d_{mx}\mathbf{e}_{415} - d_{my}\mathbf{e}_{425} - d_{mz}\mathbf{e}_{435} - d_{px}\mathbf{e}_{235} - d_{py}\mathbf{e}_{315} - d_{pz}\mathbf{e}_{125}$
Carrier	$\operatorname{car}(\mathbf{d}) = \mathbf{d}\wedge\mathbf{e}_5 = d_{vx}\mathbf{e}_{415} + d_{vy}\mathbf{e}_{425} + d_{vz}\mathbf{e}_{435} + d_{mx}\mathbf{e}_{235} + d_{my}\mathbf{e}_{315} + d_{mz}\mathbf{e}_{125}$ (flat line)
Anticarrier	$\operatorname{acr}(\mathbf{d}) = \mathbf{d}^*\wedge\mathbf{e}_5 = -d_{vx}\mathbf{e}_{4235} - d_{vy}\mathbf{e}_{4315} - d_{vz}\mathbf{e}_{4125} + d_{pw}\mathbf{e}_{3215}$ (flat plane)
Container	$\operatorname{con}(\mathbf{d}) = \operatorname{car}(\mathbf{d})^*\wedge\mathbf{d} = -(d_{vx}^2 + d_{vy}^2 + d_{vz}^2)\mathbf{e}_{1234} + (d_{vy}d_{mz} - d_{vz}d_{my} + d_{vx}d_{pw})\mathbf{e}_{4235} + (d_{vz}d_{mx} - d_{vx}d_{mz} + d_{vy}d_{pw})\mathbf{e}_{4315} + (d_{vx}d_{my} - d_{vy}d_{mx} + d_{vz}d_{pw})\mathbf{e}_{4125} - (d_{mx}^2 + d_{my}^2 + d_{mz}^2 + d_{vx}d_{px} + d_{vy}d_{py} + d_{vz}d_{pz})\mathbf{e}_{3215}$
Round Bulk	$\mathbf{d}_\bullet = d_{mx}\mathbf{e}_{23} + d_{my}\mathbf{e}_{31} + d_{mz}\mathbf{e}_{12}$
Round Weight	$\mathbf{d}_\circ = d_{vx}\mathbf{e}_{41} + d_{vy}\mathbf{e}_{42} + d_{vz}\mathbf{e}_{43}$
Attitude	$\operatorname{att}(\mathbf{d}) = \mathbf{d}\vee\mathbf{e}_{3215} = d_{vx}\mathbf{e}_1 + d_{vy}\mathbf{e}_2 + d_{vz}\mathbf{e}_3 + d_{pw}\mathbf{e}_5$
Flat Bulk	$\mathbf{d}_\blacksquare = d_{px}\mathbf{e}_{15} + d_{py}\mathbf{e}_{25} + d_{pz}\mathbf{e}_{35}$
Flat Weight	$\mathbf{d}_\square = d_{pw}\mathbf{e}_{45}$
Center Norm	$\dfrac{\lVert\mathbf{d}\rVert_\odot}{\lVert\mathbf{d}\rVert_\circ} = \sqrt{\dfrac{d_{mx}^2 + d_{my}^2 + d_{mz}^2 + d_{pw}^2}{d_{vx}^2 + d_{vy}^2 + d_{vz}^2}}$
Radius Norm	$\dfrac{\lVert\mathbf{d}\rVert_\odot}{\lVert\mathbf{d}\rVert_\circ} = \sqrt{\dfrac{d_{pw}^2 - d_{mx}^2 - d_{my}^2 - d_{mz}^2 - 2(d_{px}d_{vx} + d_{py}d_{vy} + d_{pz}d_{vz})}{d_{vx}^2 + d_{vy}^2 + d_{vz}^2}}$

Circle c (Trivector) 2D

$$\mathbf{c} = n_x\mathbf{e}_{423} + n_y\mathbf{e}_{431} + n_z\mathbf{e}_{412} + (p_y n_z - p_z n_y)\mathbf{e}_{415} + (p_z n_x - p_x n_z)\mathbf{e}_{425} + (p_x n_y - p_y n_x)\mathbf{e}_{435} + (\mathbf{p}\cdot\mathbf{n})(p_x\mathbf{e}_{235} + p_y\mathbf{e}_{315} + p_z\mathbf{e}_{125} - \mathbf{e}_{321}) - \frac{p^2 - r^2}{2}(n_x\mathbf{e}_{235} + n_y\mathbf{e}_{315} + n_z\mathbf{e}_{125})$$

$\mathbf{p} = (p_x, p_y, p_z)$ = center

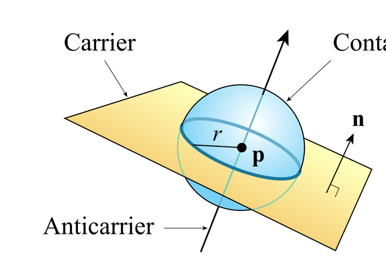
$\mathbf{n} = (n_x, n_y, n_z)$ = normal

r = radius

Anticarrier Direction; Anticarrier Moment

$$\mathbf{c} = \underbrace{c_{gx}\mathbf{e}_{423} + c_{gy}\mathbf{e}_{431} + c_{gz}\mathbf{e}_{412} + c_{gw}\mathbf{e}_{321}}_{\text{Carrier Plane}} + \underbrace{c_{vx}\mathbf{e}_{415} + c_{vy}\mathbf{e}_{425} + c_{vz}\mathbf{e}_{435} + c_{mx}\mathbf{e}_{235} + c_{my}\mathbf{e}_{315} + c_{mz}\mathbf{e}_{125}}_{\text{Flat Line}}$$

Flat Line (when $c_{gx} = c_{gy} = c_{gz} = c_{gw} = 0$)



$$c_{gy}c_{mz} - c_{gz}c_{my} - c_{gw}c_{vx} = 0$$
$$c_{gz}c_{mx} - c_{gx}c_{mz} - c_{gw}c_{vy} = 0$$
$$c_{gx}c_{my} - c_{gy}c_{mx} - c_{gw}c_{vz} = 0$$
$$c_{vx}c_{mx} + c_{vy}c_{my} + c_{vz}c_{mz} = 0$$
$$c_{gx}c_{vx} + c_{gy}c_{vy} + c_{gz}c_{vz} = 0$$

Center	$\operatorname{cen}(\mathbf{c}) = -\operatorname{acr}(\mathbf{c})\vee\mathbf{c} = (c_{gy}c_{vz} - c_{gz}c_{vy} - c_{gx}c_{gw})\mathbf{e}_1 + (c_{gz}c_{vx} - c_{gx}c_{vz} - c_{gy}c_{gw})\mathbf{e}_2 + (c_{gx}c_{vy} - c_{gy}c_{vx} - c_{gz}c_{gw})\mathbf{e}_3 + (c_{gx}^2 + c_{gy}^2 + c_{gz}^2)\mathbf{e}_4 + (c_{vx}^2 + c_{vy}^2 + c_{vz}^2 + c_{gx}c_{mx} + c_{gy}c_{my} + c_{gz}c_{mz})\mathbf{e}_5$
Dual	$\mathbf{c}^* = c_{gx}\mathbf{e}_{41} + c_{gy}\mathbf{e}_{42} + c_{gz}\mathbf{e}_{43} + c_{vx}\mathbf{e}_{23} + c_{vy}\mathbf{e}_{31} + c_{vz}\mathbf{e}_{12} + c_{mx}\mathbf{e}_{15} + c_{my}\mathbf{e}_{25} + c_{mz}\mathbf{e}_{35} - c_{gw}\mathbf{e}_{45}$
Carrier	$\operatorname{car}(\mathbf{c}) = \mathbf{c}\wedge\mathbf{e}_5 = c_{gx}\mathbf{e}_{4235} + c_{gy}\mathbf{e}_{4315} + c_{gz}\mathbf{e}_{4125} + c_{gw}\mathbf{e}_{3215}$ (flat plane)
Anticarrier	$\operatorname{acr}(\mathbf{c}) = \mathbf{c}^*\wedge\mathbf{e}_5 = c_{gx}\mathbf{e}_{415} + c_{gy}\mathbf{e}_{425} + c_{gz}\mathbf{e}_{435} + c_{vx}\mathbf{e}_{235} + c_{vy}\mathbf{e}_{315} + c_{vz}\mathbf{e}_{125}$ (flat line)
Container	$\operatorname{con}(\mathbf{c}) = \operatorname{car}(\mathbf{c})^*\wedge\mathbf{c} = -(c_{gx}^2 + c_{gy}^2 + c_{gz}^2)\mathbf{e}_{1234} + (c_{gy}c_{vz} - c_{gz}c_{vy} - c_{gx}c_{gw})\mathbf{e}_{4235} + (c_{gz}c_{vx} - c_{gx}c_{vz} - c_{gy}c_{gw})\mathbf{e}_{4315} + (c_{gx}c_{vy} - c_{gy}c_{vx} - c_{gz}c_{gw})\mathbf{e}_{4125} + (c_{gx}c_{mx} + c_{gy}c_{my} + c_{gz}c_{mz} - c_{gw}^2)\mathbf{e}_{3215}$
Round Bulk	$\mathbf{c}_\bullet = c_{gw}\mathbf{e}_{321}$
Round Weight	$\mathbf{c}_\circ = c_{gx}\mathbf{e}_{423} + c_{gy}\mathbf{e}_{431} + c_{gz}\mathbf{e}_{412}$
Attitude	$\operatorname{att}(\mathbf{c}) = \mathbf{c}\vee\mathbf{e}_{3215} = c_{gx}\mathbf{e}_{23} + c_{gy}\mathbf{e}_{31} + c_{gz}\mathbf{e}_{12} + c_{vx}\mathbf{e}_{15} + c_{vy}\mathbf{e}_{25} + c_{vz}\mathbf{e}_{35}$
Flat Bulk	$\mathbf{c}_\blacksquare = c_{mx}\mathbf{e}_{235} + c_{my}\mathbf{e}_{315} + c_{mz}\mathbf{e}_{125}$
Flat Weight	$\mathbf{c}_\square = c_{vx}\mathbf{e}_{415} + c_{vy}\mathbf{e}_{425} + c_{vz}\mathbf{e}_{435}$
Center Norm	$\dfrac{\lVert\mathbf{c}\rVert_\odot}{\lVert\mathbf{c}\rVert_\circ} = \sqrt{\dfrac{c_{gw}^2 + c_{vx}^2 + c_{vy}^2 + c_{vz}^2}{c_{gx}^2 + c_{gy}^2 + c_{gz}^2}}$
Radius Norm	$\dfrac{\lVert\mathbf{c}\rVert_\odot}{\lVert\mathbf{c}\rVert_\circ} = \sqrt{\dfrac{c_{vx}^2 + c_{vy}^2 + c_{vz}^2 - c_{gw}^2 + 2(c_{gx}c_{mx} + c_{gy}c_{my} + c_{gz}c_{mz})}{c_{gx}^2 + c_{gy}^2 + c_{gz}^2}}$

By Eric Lengyel
Copyright © 2023