

Conformal Geometric Algebra

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JOIN

MEET

Join Operation	Illustration
Dipole containing round points \mathbf{a} and \mathbf{b} .	
$\mathbf{a} \wedge \mathbf{b} = (\mathbf{a}_x \mathbf{b}_z - \mathbf{a}_z \mathbf{b}_x) \mathbf{e}_{41} + (\mathbf{a}_y \mathbf{b}_z - \mathbf{a}_z \mathbf{b}_y) \mathbf{e}_{42} + (\mathbf{a}_x \mathbf{b}_y - \mathbf{a}_y \mathbf{b}_x) \mathbf{e}_{43}$ + $(\mathbf{a}_1 \mathbf{b}_2 - \mathbf{a}_2 \mathbf{b}_1) \mathbf{e}_{23} + (\mathbf{a}_2 \mathbf{b}_3 - \mathbf{a}_3 \mathbf{b}_2) \mathbf{e}_{13} + (\mathbf{a}_1 \mathbf{b}_3 - \mathbf{a}_3 \mathbf{b}_1) \mathbf{e}_{12}$ + $(\mathbf{a}_2 \mathbf{b}_1 - \mathbf{a}_1 \mathbf{b}_2) \mathbf{e}_{35} + (\mathbf{a}_1 \mathbf{b}_2 - \mathbf{a}_2 \mathbf{b}_1) \mathbf{e}_{25} + (\mathbf{a}_2 \mathbf{b}_3 - \mathbf{a}_3 \mathbf{b}_2) \mathbf{e}_{45}$	
Line containing flat point \mathbf{p} and round point \mathbf{a} .	
$\mathbf{p} \wedge \mathbf{a} = (\mathbf{p}_x \mathbf{a}_w - \mathbf{p}_w \mathbf{a}_x) \mathbf{e}_{415} + (\mathbf{p}_y \mathbf{a}_x - \mathbf{p}_x \mathbf{a}_y) \mathbf{e}_{315}$ + $(\mathbf{p}_z \mathbf{a}_w - \mathbf{p}_w \mathbf{a}_z) \mathbf{e}_{425} + (\mathbf{p}_z \mathbf{a}_x - \mathbf{p}_x \mathbf{a}_z) \mathbf{e}_{325}$ + $(\mathbf{p}_z \mathbf{a}_w - \mathbf{p}_w \mathbf{a}_z) \mathbf{e}_{435} + (\mathbf{p}_y \mathbf{a}_x - \mathbf{p}_x \mathbf{a}_y) \mathbf{e}_{125}$	
Circle containing dipole \mathbf{d} and round point \mathbf{a} .	
$\mathbf{d} \wedge \mathbf{a} = (\mathbf{d}_x \mathbf{a}_z - \mathbf{d}_z \mathbf{a}_x + \mathbf{d}_{mx} \mathbf{a}_w) \mathbf{e}_{423} + (\mathbf{d}_x \mathbf{a}_z - \mathbf{d}_{mx} \mathbf{a}_x + \mathbf{d}_{my} \mathbf{a}_w) \mathbf{e}_{431}$ + $(\mathbf{d}_y \mathbf{a}_z - \mathbf{d}_z \mathbf{a}_y + \mathbf{d}_{mx} \mathbf{a}_x) \mathbf{e}_{412} + (\mathbf{d}_{mx} \mathbf{a}_z - \mathbf{d}_{my} \mathbf{a}_x + \mathbf{d}_{mz} \mathbf{a}_w) \mathbf{e}_{321}$ + $(\mathbf{d}_x \mathbf{a}_y - \mathbf{d}_{py} \mathbf{a}_x + \mathbf{d}_{py} \mathbf{a}_z) \mathbf{e}_{415} + (\mathbf{d}_y \mathbf{a}_x - \mathbf{d}_{py} \mathbf{a}_z + \mathbf{d}_{py} \mathbf{a}_x) \mathbf{e}_{425}$ + $(\mathbf{d}_z \mathbf{a}_x - \mathbf{d}_{py} \mathbf{a}_y + \mathbf{d}_{py} \mathbf{a}_z) \mathbf{e}_{435} + (\mathbf{d}_{mx} \mathbf{a}_x - \mathbf{d}_{pz} \mathbf{a}_z + \mathbf{d}_{mz} \mathbf{a}_x) \mathbf{e}_{315}$ + $(\mathbf{d}_{mx} \mathbf{a}_x - \mathbf{d}_{pz} \mathbf{a}_z + \mathbf{d}_{mz} \mathbf{a}_x) \mathbf{e}_{445} + (\mathbf{d}_y \mathbf{a}_x - \mathbf{d}_{pz} \mathbf{a}_y + \mathbf{d}_{py} \mathbf{a}_x) \mathbf{e}_{125}$	
Plane containing line \mathbf{l} and round point \mathbf{a} .	
$\mathbf{l} \wedge \mathbf{a} = (\mathbf{l}_x \mathbf{a}_z - \mathbf{l}_y \mathbf{a}_x - \mathbf{l}_{mx} \mathbf{a}_w) \mathbf{e}_{4235} + (\mathbf{l}_x \mathbf{a}_z - \mathbf{l}_z \mathbf{a}_x - \mathbf{l}_{wy} \mathbf{a}_w) \mathbf{e}_{4315}$ + $(\mathbf{l}_y \mathbf{a}_x - \mathbf{l}_z \mathbf{a}_y - \mathbf{l}_{mx} \mathbf{a}_w) \mathbf{e}_{4215} + (\mathbf{l}_{mx} \mathbf{a}_x - \mathbf{l}_{my} \mathbf{a}_y + \mathbf{l}_{mz} \mathbf{a}_z) \mathbf{e}_{3215}$	
Plane containing dipole \mathbf{d} and flat point \mathbf{p} .	
$\mathbf{d} \wedge \mathbf{p} = (\mathbf{d}_x \mathbf{p}_z - \mathbf{d}_z \mathbf{p}_x + \mathbf{d}_{mx} \mathbf{p}_w) \mathbf{e}_{4235}$ + $(\mathbf{d}_y \mathbf{p}_z - \mathbf{d}_z \mathbf{p}_y + \mathbf{d}_{my} \mathbf{p}_w) \mathbf{e}_{4315}$ + $(\mathbf{d}_z \mathbf{p}_x - \mathbf{d}_y \mathbf{p}_x + \mathbf{d}_{my} \mathbf{p}_z) \mathbf{e}_{4125}$ - $(\mathbf{d}_{mx} \mathbf{p}_x + \mathbf{d}_{my} \mathbf{p}_y + \mathbf{d}_{mz} \mathbf{p}_z) \mathbf{e}_{215}$	
Sphere containing circle \mathbf{c} and round point \mathbf{a} .	
$\mathbf{c} \wedge \mathbf{a} = -(\mathbf{c}_x \mathbf{a}_z + \mathbf{c}_y \mathbf{a}_y + \mathbf{c}_z \mathbf{a}_w) \mathbf{e}_{1234}$ + $(\mathbf{c}_x \mathbf{a}_y - \mathbf{c}_y \mathbf{a}_x + \mathbf{c}_w \mathbf{a}_w) \mathbf{e}_{1235}$ + $(\mathbf{c}_x \mathbf{a}_z - \mathbf{c}_y \mathbf{a}_x + \mathbf{c}_w \mathbf{a}_w) \mathbf{e}_{1315}$ + $(\mathbf{c}_y \mathbf{a}_x + \mathbf{c}_z \mathbf{a}_y + \mathbf{c}_w \mathbf{a}_w) \mathbf{e}_{1425}$ + $(\mathbf{c}_y \mathbf{a}_z - \mathbf{c}_z \mathbf{a}_x + \mathbf{c}_w \mathbf{a}_w) \mathbf{e}_{1215}$ + $(\mathbf{c}_z \mathbf{a}_x + \mathbf{c}_w \mathbf{a}_y + \mathbf{c}_w \mathbf{a}_w) \mathbf{e}_{1325}$	
Sphere containing dipoles \mathbf{d} and \mathbf{f} .	
$\mathbf{d} \wedge \mathbf{f} = -(\mathbf{d}_x \mathbf{f}_w + \mathbf{d}_y \mathbf{f}_w + \mathbf{d}_{mx} \mathbf{f}_z + \mathbf{d}_{my} \mathbf{f}_z + \mathbf{d}_{mz} \mathbf{f}_w) \mathbf{e}_{1234}$ + $(\mathbf{d}_x \mathbf{f}_z - \mathbf{d}_z \mathbf{f}_y + \mathbf{d}_p \mathbf{f}_y + \mathbf{d}_{py} \mathbf{f}_z + \mathbf{d}_{pz} \mathbf{f}_w) \mathbf{e}_{1235}$ + $(\mathbf{d}_y \mathbf{f}_z - \mathbf{d}_z \mathbf{f}_x + \mathbf{d}_p \mathbf{f}_x + \mathbf{d}_{px} \mathbf{f}_y + \mathbf{d}_{py} \mathbf{f}_z) \mathbf{e}_{4235}$ + $(\mathbf{d}_{mx} \mathbf{f}_z - \mathbf{d}_z \mathbf{f}_{mx} + \mathbf{d}_p \mathbf{f}_{mx} + \mathbf{d}_{px} \mathbf{f}_z + \mathbf{d}_{py} \mathbf{f}_{mx}) \mathbf{e}_{4315}$ + $(\mathbf{d}_{my} \mathbf{f}_z - \mathbf{d}_z \mathbf{f}_{my} + \mathbf{d}_p \mathbf{f}_{my} + \mathbf{d}_{px} \mathbf{f}_z + \mathbf{d}_{py} \mathbf{f}_{my}) \mathbf{e}_{4215}$ - $(\mathbf{d}_{mz} \mathbf{f}_z - \mathbf{d}_z \mathbf{f}_{mz} + \mathbf{d}_p \mathbf{f}_{mz} + \mathbf{d}_{px} \mathbf{f}_z + \mathbf{d}_{py} \mathbf{f}_{mz}) \mathbf{e}_{3215}$	

EXPANSION	
Expansion Operation	Illustration
Dipole containing round point \mathbf{a} and orthogonal to sphere \mathbf{s} .	
$\mathbf{a} \wedge \mathbf{s}^{\circ} = (\mathbf{a}_x \mathbf{s}_y + \mathbf{a}_y \mathbf{s}_x) \mathbf{e}_{41} + (\mathbf{a}_z \mathbf{s}_x - \mathbf{a}_x \mathbf{s}_z) \mathbf{e}_{23}$ + $(\mathbf{a}_x \mathbf{s}_y + \mathbf{a}_w \mathbf{s}_y) \mathbf{e}_{31} + (\mathbf{a}_z \mathbf{s}_x + \mathbf{a}_w \mathbf{s}_x) \mathbf{e}_{423}$ + $(\mathbf{a}_z \mathbf{s}_x - \mathbf{a}_y \mathbf{s}_z) \mathbf{e}_{12} + (\mathbf{a}_x \mathbf{s}_y - \mathbf{a}_z \mathbf{s}_y) \mathbf{e}_{235}$ - $(\mathbf{a}_x \mathbf{s}_w + \mathbf{a}_w \mathbf{s}_x) \mathbf{e}_{35} + (\mathbf{a}_z \mathbf{s}_w - \mathbf{a}_w \mathbf{s}_w) \mathbf{e}_{45}$	
Expansion Operation	Illustration
Sphere containing circle \mathbf{c} and orthogonal to sphere \mathbf{s} .	
$\mathbf{c} \wedge \mathbf{s}^{\circ} = (\mathbf{c}_{px} \mathbf{s}_y - \mathbf{c}_y \mathbf{s}_{px}) \mathbf{e}_{1234}$ + $(\mathbf{c}_{px} \mathbf{s}_y + \mathbf{c}_y \mathbf{s}_{px}) \mathbf{e}_{4235}$ + $(\mathbf{c}_{px} \mathbf{s}_z - \mathbf{c}_z \mathbf{s}_{px}) \mathbf{e}_{1315}$ + $(\mathbf{c}_{py} \mathbf{s}_x - \mathbf{c}_x \mathbf{s}_{py}) \mathbf{e}_{1425}$ + $(\mathbf{c}_{py} \mathbf{s}_x + \mathbf{c}_x \mathbf{s}_{py}) \mathbf{e}_{3215}$ - $(\mathbf{c}_{pz} \mathbf{s}_x + \mathbf{c}_x \mathbf{s}_{pz}) \mathbf{e}_{435} + (\mathbf{c}_{pz} \mathbf{s}_w - \mathbf{c}_w \mathbf{s}_{pz}) \mathbf{e}_{3325}$	
Expansion Operation	Illustration
Dipole containing round point \mathbf{a} and orthogonal to plane \mathbf{g} .	
$\mathbf{a} \wedge \mathbf{g}^{\circ} = (\mathbf{a}_x \mathbf{g}_z + \mathbf{a}_z \mathbf{g}_x) \mathbf{e}_{41} + (\mathbf{a}_y \mathbf{g}_z - \mathbf{a}_z \mathbf{g}_y) \mathbf{e}_{31}$ + $(\mathbf{a}_x \mathbf{g}_z + \mathbf{a}_w \mathbf{g}_z) \mathbf{e}_{4235}$ + $(\mathbf{a}_y \mathbf{g}_z - \mathbf{a}_y \mathbf{g}_w) \mathbf{e}_{4125}$ - $(\mathbf{a}_x \mathbf{g}_w + \mathbf{a}_w \mathbf{g}_x) \mathbf{e}_{35} + (\mathbf{a}_z \mathbf{g}_w - \mathbf{a}_w \mathbf{g}_z) \mathbf{e}_{45}$	
Expansion Operation	Illustration
Sphere containing circle \mathbf{c} and orthogonal to plane \mathbf{g} .	
$\mathbf{c} \wedge \mathbf{g}^{\circ} = -(\mathbf{c}_{px} \mathbf{g}_z - \mathbf{c}_z \mathbf{g}_{px}) \mathbf{e}_{1234}$ + $(\mathbf{c}_{px} \mathbf{g}_z + \mathbf{c}_z \mathbf{g}_{px}) \mathbf{e}_{4235}$ + $(\mathbf{c}_{py} \mathbf{g}_z - \mathbf{c}_z \mathbf{g}_{py}) \mathbf{e}_{1315}$ + $(\mathbf{c}_{py} \mathbf{g}_z + \mathbf{c}_z \mathbf{g}_{py}) \mathbf{e}_{4125}$ + $(\mathbf{c}_{pz} \mathbf{g}_z - \mathbf{c}_z \mathbf{g}_{pz}) \mathbf{e}_{3215}$ - $(\mathbf{c}_{px} \mathbf{g}_w - \mathbf{c}_w \mathbf{g}_{px}) \mathbf{e}_{435} + (\mathbf{c}_{py} \mathbf{g}_w - \mathbf{c}_w \mathbf{g}_{py}) \mathbf{e}_{3325}$	
Expansion Operation	Illustration
Plane containing line \mathbf{l} and orthogonal to sphere \mathbf{s} .	
$\mathbf{l} \wedge \mathbf{s}^{\circ} = (\mathbf{l}_{px} \mathbf{s}_y - \mathbf{l}_y \mathbf{s}_{px}) \mathbf{e}_{4235} + (\mathbf{l}_{px} \mathbf{s}_z - \mathbf{l}_z \mathbf{s}_{px}) \mathbf{e}_{1315}$ + $(\mathbf{l}_{py} \mathbf{s}_x - \mathbf{l}_x \mathbf{s}_{py}) \mathbf{e}_{1425} + (\mathbf{l}_{py} \mathbf{s}_z - \mathbf{l}_z \mathbf{s}_{py}) \mathbf{e}_{3215}$ + $(\mathbf{l}_{pz} \mathbf{s}_x - \mathbf{l}_x \mathbf{s}_{pz}) \mathbf{e}_{435} + (\mathbf{l}_{pz} \mathbf{s}_y - \mathbf{l}_y \mathbf{s}_{pz}) \mathbf{e}_{3325}$	
Expansion Operation	Illustration
Circle containing dipole \mathbf{d} and orthogonal to plane \mathbf{g} .	
$\mathbf{d} \wedge \mathbf{g}^{\circ} = (\mathbf{d}_x \mathbf{g}_z - \mathbf{d}_z \mathbf{g}_x) \mathbf{e}_{41} + (\mathbf{d}_y \mathbf{g}_z - \mathbf{d}_z \mathbf{g}_y) \mathbf{e}_{31}$ + $(\mathbf{d}_x \mathbf{g}_z + \mathbf{d}_w \mathbf{g}_z) \mathbf{e}_{4235}$ + $(\mathbf{d}_y \mathbf{g}_z - \mathbf{d}_y \mathbf{g}_w) \mathbf{e}_{4125}$ - $(\mathbf{d}_x \mathbf{g}_w + \mathbf{d}_w \mathbf{g}_x) \mathbf{e}_{35} + (\mathbf{d}_z \mathbf{g}_w - \mathbf{d}_z \mathbf{g}_x) \mathbf{e}_{45}$	
Expansion Operation	Illustration
Plane containing line \mathbf{l} and orthogonal to sphere \mathbf{s} .	
$\mathbf{l} \wedge \mathbf{s}^{\circ} = (\mathbf{l}_{px} \mathbf{s}_y - \mathbf{l}_y \mathbf{s}_{px}) \mathbf{e}_{4235} + (\mathbf{l}_{px} \mathbf{s}_z - \mathbf{l}_z \mathbf{s}_{px}) \mathbf{e}_{1315}$ + $(\mathbf{l}_{py} \mathbf{s}_x - \mathbf{l}_x \mathbf{s}_{py}) \mathbf{e}_{1425} + (\mathbf{l}_{py} \mathbf{s}_z - \mathbf{l}_z \mathbf{s}_{py}) \mathbf{e}_{3215}$ + $(\mathbf{l}_{pz} \mathbf{s}_x - \mathbf{l}_x \mathbf{s}_{pz}) \mathbf{e}_{435} + (\mathbf{l}_{pz} \mathbf{s}_y - \mathbf{l}_y \mathbf{s}_{pz}) \mathbf{e}_{3325}$	
Expansion Operation	Illustration
Circle containing dipole \mathbf{d} and orthogonal to plane \mathbf{g} .	
$\mathbf{d} \wedge \mathbf{g}^{\circ} = (\mathbf{d}_x \mathbf{g}_z - \mathbf{d}_z \mathbf{g}_x) \mathbf{e}_{41} + (\mathbf{d}_y \mathbf{g}_z - \mathbf{d}_z \mathbf{g}_y) \mathbf{e}_{31}$ + $(\mathbf{d}_x \mathbf{g}_z + \mathbf{d}_w \mathbf{g}_z) \mathbf{e}_{4235}$ + $(\mathbf{d}_y \mathbf{g}_z - \mathbf{d}_y \mathbf{g}_w) \mathbf{e}_{4125}$ - $(\mathbf{d}_x \mathbf{g}_w + \mathbf{d}_w \mathbf{g}_x) \mathbf{e}_{35} + (\mathbf{d}_z \mathbf{g}_w - \mathbf{d}_z \mathbf{g}_x) \mathbf{e}_{45}$	